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MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2022/2023 (TERM ID 2210)

PEM0026 – TRIGONOMETRY & GEOMETRY

(Foundation in Engineering)

17 FEBRUARY 2023
9.00 a.m – 11.00 a.m
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **SIX (6)** printed pages including the cover page and appendix.
2. Attempt **ALL FOUR** questions. All questions carry equal marks and the distribution of the marks for each question is given.
3. Please write all your answers in the answer booklet provided. All necessary working steps **MUST** be shown.

APPENDIX**Formula****Trigonometry Identities**

$$\sin(-\theta) = -\sin \theta$$

$$\cos(-\theta) = \cos \theta$$

$$\tan(-\theta) = -\tan \theta$$

$$\csc(-\theta) = -\csc \theta$$

$$\sec(-\theta) = \sec \theta$$

$$\cot(-\theta) = -\cot \theta$$

$$\sin(\theta) = \cos(90^\circ - \theta)$$

$$\csc(\theta) = \sec(90^\circ - \theta)$$

$$\tan(\theta) = \cot(90^\circ - \theta)$$

$$\cos(\theta) = \sin(90^\circ - \theta)$$

$$\sec(\theta) = \csc(90^\circ - \theta)$$

$$\cot(\theta) = \tan(90^\circ - \theta)$$

$$\cos^2 A + \sin^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\csc^2 A = 1 + \cot^2 A$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$= 2 \cos^2 A - 1$$

$$= 1 - 2 \sin^2 A$$

$$\sin 2A = 2 \sin A \cos A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

$$\sin A \cos B = \frac{1}{2} [\sin(A+B) + \sin(A-B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A+B) + \cos(A-B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A+B) - \sin(A-B)]$$

$$\sin A \sin B = \frac{1}{2} [\cos(A-B) - \cos(A+B)]$$

$$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\sin A - \sin B = 2 \cos \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\cos A + \cos B = 2 \cos \frac{A+B}{2} \cos \frac{A-B}{2}$$

$$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$$

$$\sin \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{2}}$$

$$\cos \frac{A}{2} = \pm \sqrt{\frac{1 + \cos A}{2}}$$

$$\tan \frac{A}{2} = \pm \sqrt{\frac{1 - \cos A}{1 + \cos A}} = \frac{1 - \cos A}{\sin A} = \frac{\sin A}{1 + \cos A}$$

$$\sin \theta = \cos(90^\circ - \theta)$$

$$\cos \theta = \sin(90^\circ - \theta)$$

$$\tan \theta = \cot(90^\circ - \theta)$$

$$\csc \theta = \sec(90^\circ - \theta)$$

$$\sec \theta = \csc(90^\circ - \theta)$$

$$\cot \theta = \tan(90^\circ - \theta)$$

$$z_k = \sqrt[n]{r} \left[\cos \left(\frac{\theta}{n} + \frac{360^\circ k}{n} \right) + i \sin \left(\frac{\theta}{n} + \frac{360^\circ k}{n} \right) \right], \text{ where } k = 0, 1, 2, \dots, n-1$$

QUESTION 1 [25 marks]

- (a) Find the exact value each of the expression below by using the Complementary Angle Theorem. Do not use calculator.

(i) $\tan 20^\circ - \frac{\cos 70^\circ}{\cos 20^\circ}$ (2 marks)

(ii) $\sin^2 65^\circ + \sin^2 25^\circ$ (2 marks)

- (b) Given the function: $y = 2 \sin\left(\frac{1}{3}x + \pi\right) + 4$, $-3\pi \leq x \leq 3\pi$.

- (i) Determine the amplitude, period and phase shift of the function. (5 marks)

- (ii) Graph the function for 1 cycle. State clearly the maximum and minimum points. (6 marks)

- (c) The area of the sector of a circle of radius r formed by the central angle 60° is 6π meter². Find the value of r . (4 marks)

- (d) Given that $\sin \theta = \frac{a}{5}$, $\tan \theta = \frac{a}{1+a}$ and θ lies in the first quadrant.

- (i) Find the constant value of a . (4 marks)

- (ii) Find the exact value of $\sec \theta$. (2 marks)

Continued...

QUESTION 2 [25 marks]

- (a) Find the exact value of $\tan \left[\sin^{-1} \left(\frac{1}{2} \right) \right]$. (3 marks)
- (b) Establish the following identities:
- (i) $\tan \theta \cot \theta - \cos^2 \theta = \sin^2 \theta$. (2 marks)
- (ii) $\frac{\tan \theta + \sin \theta}{1 + \cos \theta} = \tan \theta$. (4 marks)
- (c) Given that $\sin \alpha = \frac{5}{6}$ and $\cos \beta = \frac{2}{7}$, where $90^\circ \leq \alpha \leq 180^\circ$ and $-90^\circ \leq \beta \leq 0^\circ$. Find the exact value of
- (i) $\sin (\alpha - \beta)$. (6 marks)
- (ii) $\cos (\alpha + \beta)$. (2 marks)
- (iii) $\tan 2\alpha$. (2 marks)
- (d) Solve the trigonometric equation:
- $$2 \sin^2 \theta - 3 \sin \theta + 1 = 0 \quad \text{where } 0^\circ \leq \theta \leq 360^\circ \quad (6 \text{ marks})$$

Continued...

QUESTION 3 [25 marks]

- (a) Given $z = -5 - 2i$ and $w = -2 + 2i$. Find
- (i) the polar form of w and z . Leave your answer in 2 decimal places. (7 marks)
 - (ii) zw in polar form. (2 marks)
 - (iii) $\frac{z}{w}$ in polar form. (2 marks)
 - (iv) $(zw)^4$ in polar form. (2 marks)
- (b) Given that triangle PQR on a 3D space has vertices at the points $P(1, 0, -1)$, $Q(2, 4, 5)$ and $R(3, 1, 7)$.
- (i) Find the vectors $\vec{PQ}$ and $\vec{PR}$. (2 marks)
 - (ii) Find the angle between vectors $\vec{PQ}$ and $\vec{PR}$. (5 marks)
- (c) Find the area of parallelogram whose adjacent sides are determined by the vectors $\vec{u} = i - j + k$ and $\vec{v} = 3i + j + 4k$. (5 marks)

Continued...

QUESTION 4 [25 marks]

- (a) (i) Find the equation of a straight line that passes through the point $(3, -2)$, and it is parallel to another line L with an equation of $2x - 5y - 9 = 0$. (4 marks)
- (ii) Given that the lines $3x + ky + 2 = 0$ and $4x + y - 1 = 0$ are perpendicular. Find the value of k . (4 marks)
- (iii) Find the midpoint of the line segment from $A(1,1)$ to $B(3,4)$. (2 marks)
- (b) Find the vertex, focus and directrix of the parabola $(y - 4)^2 = 4(x + 1)$. (4 marks)
- (c) Given that the equation of an ellipse is $9x^2 + 4y^2 - 18x + 8y - 23 = 0$, find the center, foci and vertices of the ellipse. Then, graph the ellipse. (11 marks)

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